# On Fuzzy Real-valued Multiple Sequence Spaces ${}_3\ell^F(p)$

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**Abstract: The purpose of this article is to extend a generalized convergence method to sequence $\ell(p)$ of fuzzy numbers of multiplicity greater than two. Here we introduce the classes of fuzzy real-valued multiple sequences ${}_3\ell^F(p)$ where $p=\langle p_{nkl}\rangle$ is a triple sequence of bounded strictly positive numbers. We study different topological properties like completeness, solidness, symmetricity, convergence free etc of this space. We prove some inclusion results also.**

**Keywords: Fuzzy real valued triple sequence, Multiple sequences, Solid, Monotone, Symmetric, Convergence free, Sequence algebra etc.**

## I. INTRODUCTION

Fuzzy set theory, compared to other mathematical theories, is perhaps the most easily adaptable theory to practice. Instead of defining an entity in calculus by assuming that its role is exactly known, we can use fuzzy sets to define the same entity by allowing possible deviations and inexactness in its role. This representation suits well the uncertainties encountered in practical life, which make fuzzy sets a valuable mathematical tool. The concepts of fuzzy sets and fuzzy set operations were first introduced by Zadeh [25] and subsequently several authors have discussed various aspects of the theory and applications of fuzzy sets. In fact the fuzzy set theory has become an area of active area of research in science and engineering for the last 40 years. Fuzzy set theory is a powerful hand set for modelling uncertainty and vagueness in various problems arising in the field of science and engineering. It extends the scope and results of classical mathematical analysis by applying fuzzy logic to conventional mathematical objects, such as functions, sequences and series etc. The ideas of fuzzy set theory have been used widely not only in many engineering applications, such as, computer programming [9], quantum physics [15], control of chaos [8], bifurcation of non-linear dynamical system [11] etc., but also in various branches of mathematics, such as, theory of metric and topological spaces [6], theory of linear systems [20], studies of convergence of sequences of functions ([3],[12]). While studying fuzzy topological spaces, we face many situations where we need to deal with convergence of fuzzy numbers.

Using the notion of fuzzy real numbers, different types of fuzzy real-valued sequence spaces have been introduced and studied by several mathematicians. The initial works on double sequences of real or complex terms are found in Bromwich [2]. Hardy [10] introduced the notion of regular convergence for double sequences of real or complex terms. Agnew [1] studied the summability theory of multiple sequences and obtained certain theorems which have already been proved for double sequences by the author himself. Móricz [16] extended statistical convergence from single to multiple real sequences and obtained some results for real double sequences. Şahiner *et al.* [21] developed statistical convergence for triple sequences of real numbers. Savas and Esi [22], Esi [7] developed statistical convergence of triple sequences on probabilistic normed space. Some more works on triple sequences can be found in ([5], [14]). Nanda [18] introduced and studied fuzzy real-valued double sequence space $\ell_p^F$ for $1 \le p < \infty$. Nuray and Savas [19] have studied some properties of the space $\ell(p)^F$. Subsequently many authors have worked on the space $\ell(p)$ such as ([4],[17],[22],[23]). In this article we shall investigate the class of triple sequences ${}_3\ell^F(p)$.

## II. DEFINITIONS AND BACKGROUND

Throughout *N*, *R* and *C* denote the sets of natural, real and complex numbers respectively.

A fuzzy real number $X$ is a fuzzy set on $R$, i.e. a mapping $X : R \to L(=[0,1]$ associating each real number $t$ with its grade of membership $X(t)$. Every real number *r* can be expressed as a fuzzy real number $\bar{r}$ as follows:

$$\bar{r}(t) = \begin{cases} 1 & if \quad t = r \\ 0 & otherwise \end{cases}$$

The $\alpha$-level set of a fuzzy real number $X$, $0 < \alpha \le 1$ denoted by $[X]^\alpha$ is defined as $[X]^\alpha = \{t \in R : X(t) \ge \alpha\}$.

A fuzzy real number *X* is called convex if $X(t) \ge X(s) \wedge X(r) = \min(X(s), X(r))$, where $s<t<r$.

If there exists $t_0 \in R$ such that $X(t_0) = 1$, then the fuzzy real number $X$ is called normal. A fuzzy real number

$X$ is said to be upper semi-continuous if for each $\varepsilon > 0$, $X^{-1}[0, a+\varepsilon))$, for all $a \in L$ is open in the usual topology of *R*. The set of all upper semi continuous, normal, convex fuzzy number is denoted by *R(L)*.

Let $D$ be the set of all closed bounded intervals $X = [X^L, X^R]$ on the real line *R*. Then $X \le Y$ if and only if $X^L \le Y^L$ and $X^R \le Y^R$. Also let $d(X,Y) = \max(|X^L - X^R|, |Y^L - Y^R|)$.

Then $(D,d)$ is a complete metric space.

Let $\bar{d} : R(L) \times R(L) \to R$ be defined by

$\bar{d}(X,Y) = \sup_{0 \le \alpha \le 1} d([X]^\alpha, [Y]^\alpha)$, for $X, Y \in R(L)$.

Then $\bar{d}$ defines a metric on $R(L)$ and $(R(L), \bar{d})$ is a complete metric space.

The **Arithmetic operations** on $R(L)$ are defined as follows:

$$(X \oplus Y)(t) = \sup_{s \in R}\{X(s) \wedge Y(t-s)\}, t \in R$$

$$(X \Theta Y)(t) = \sup_{s \in R}\{X(s) \wedge Y(S-t)\}, t \in R$$

$$(X \otimes Y)(t) = \sup_{s \in R}\{X(s) \wedge Y(t/s)\}, t \in R$$

$$(X / Y)(t) = \sup_{s \in R}\{X(st) \wedge Y(s)\}, t \in R$$

A triple sequence (real or complex) can be defined as a function $x : N \times N \times N \to R(C)$.

A triple sequence of fuzzy numbers is a triple infinite array of fuzzy real numbers $X_{nkl}$ for all $n, k, l \in N$ and is denoted by $\langle X_{nkl} \rangle$ where $X_{nkl} \in R(L)$.

A triple sequence $X = \langle X_{nkl} \rangle$ of fuzzy numbers is said to be convergent to a fuzzy real number $X_0$,
if for each $\varepsilon > 0$, there exists a positive integer $m$ such that $\bar{d}(X_{nkl}, X_0) < \varepsilon$ for all $n, k, l \ge m$.

A triple sequence $X = \langle X_{nkl} \rangle$ of fuzzy numbers is said to be a Cauchy sequence, if for each $\varepsilon > 0$, there exists a positive integer $n_0$ such that $\bar{d}(X_{nkl}, X_{pqr}) < \varepsilon$ for every $n \ge p \ge n_0$, $k \ge q \ge k_0, l \ge r \ge l_0$.

A triple sequence $X = \langle X_{nkl} \rangle$ of fuzzy numbers is said to be bounded if there exists a positive integer *M* such that $\bar{d}(X_{nkl}, \bar{0}) < M$ for all *n, k, l*.

Let ${}_3\ell_\infty$ denote the set of all bounded triple sequences of fuzzy numbers which is a normed space, normed by $\|X\| = \sup_{n,k,l} |X_{nkl}|$.

A fuzzy real valued triple sequence space $E^F$ is said to be solid if $\langle Y_{nkl} \rangle \in E^F$ whenever $|Y_{nkl}| \le |X_{nkl}|$ for all $n, k, l \in N$ and $\langle X_{nkl} \rangle \in E^F$..

A fuzzy real valued triple sequence space $E^F$ is said to be monotone if $E^F$ contains the canonical pre-image of all its step spaces.

A fuzzy real valued triple sequence $E^F$ is said to be symmetric if $S(X) \subset E^F$, for all $X \in E^F$, where S(*X*) denotes the set of all permutations of the elements of $X = \langle X_{nkl} \rangle$

A fuzzy real valued triple sequence space $E^F$ is said to be sequence algebra if $\langle X_{nkl} \otimes Y_{nkl} \rangle \in E^F$, whenever $\langle X_{nkl} \rangle, \langle Y_{nkl} \rangle \in E^F$.

A fuzzy real valued triple sequence space $E^F$ is said to be convergence free if $\langle Y_{nkl} \rangle \in E^F$, whenever $\langle X_{nkl} \rangle \in E^F$ and $X_{nkl} = \bar{0}$ implies $Y_{nkl} = \bar{0}$.

Let $p = \langle p_{nkl} \rangle$ be a triple sequence of bounded strictly positive numbers. We define the following fuzzy real-valued triple sequence space:

$${}_3\ell^F(p) = \left\{ X = (X_{nkl}) : \sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left[\bar{d}(X_{nkl}, \bar{0})\right]^{p_{nkl}} < \infty \right\}.$$

**Lemma.** *If a sequence space $E^F$ is solid, then it is monotone.*

For the crisp set case, one may refer to Kamthan and Gupta [13], p.53.

## III. MAIN RESULTS

**Theorem1.** *The space ${}_3\ell^F(p)$ is a complete metric space with respect to the metric ρ defined by*

$$\rho(X,Y) = \left( \sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left[\bar{d}(X_{nkl}, Y_{nkl})\right]^{p_{nkl}} \right)^{\frac{1}{M}}, \text{ where}$$

$$M = \max(1, \sup_{nkl} p_{nkl}).$$

**Proof.** Let $\langle X^{(i)} \rangle$ be a Cauchy sequence in ${}_3\ell^F(p)$ where $X^{(i)} = \langle X_{nkl}^{(i)} \rangle$.

Then for a given $\varepsilon > 0$, there exists $n_0 \in N$ such that

$$\rho\left(X^{(i)}, X^{(j)}\right) < \varepsilon, \text{ for all } i, j \ge n_0 \qquad (1)$$

$$\Rightarrow \left( \sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left[\bar{d}(X_{nkl}^{(i)}, X_{nkl}^{(j)})\right]^{p_{nkl}} \right)^{\frac{1}{M}} < \varepsilon, \text{ for all } i, j \ge n_0.$$

$$\Rightarrow \left( \sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left[\bar{d}(X_{nkl}^{(i)}, X_{nkl}^{(j)})\right]^{p_{nkl}} \right) < \varepsilon^M, \text{ for all } i, j \ge n_0.$$

$\Rightarrow \bar{d}(X_{nkl}^{(i)}, X_{nkl}^{(j)}) < \varepsilon$, for all $i, j \ge n_0$ and for all $n, k, l \in N$.

$\Rightarrow \langle X_{nkl}^{(i)} \rangle_{i=1}^{\infty}$ is a Cauchy sequence in *R(L)* for each $n, k, l \in N$.

Since $R(L)$ is complete, so $\langle X_{nkl}^{(i)} \rangle_{i=1}^{\infty}$ is convergent for each $n, k, l \in N$.

Let $\lim_i X_{nkl}^{(i)} = X_{nkl}$, for each $n,k,l \in N$ and $X = \langle X_{nkl} \rangle$.

Taking limit as $j \to \infty$ in equation (1), we have

$$\rho\left(X^{(i)}, X\right) < \varepsilon, \text{ for all } i \geq n_0.$$

Now for all $i \geq n_0$,

$$\rho\left(X, \bar{0}\right) \leq \rho\left(X, X^{(i)}\right) + \rho\left(X^{(i)}, \bar{0}\right) \leq \varepsilon + K < \infty.$$

This implies $X \in {}_3\ell^F(p)$. Hence ${}_3\ell^F(p)$ is complete. ■

**Theorem 2.** *The space* ${}_3\ell^F(p)$ *is solid as well as monotone*.

**Proof.** Let $\langle X_{nkl} \rangle \in {}_3\ell^F(p)$ and $\langle Y_{nkl} \rangle$ be a fuzzy real valued triple sequence such that

$\bar{d}(Y_{nkl}, \bar{0}) \leq \bar{d}(X_{nkl}, \bar{0})$, for all $n,k,l \in N$.

Then $\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left[\bar{d}(X_{nkl}, \bar{0})\right]^{p_{nkl}} < \infty$

Now

$$\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left[\bar{d}(Y_{nkl}, \bar{0})\right]^{p_{nkl}} \leq \sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left[\bar{d}(X_{nkl}, \bar{0})\right]^{p_{nkl}} < \infty$$

Thus $\langle Y_{nkl} \rangle \in {}_3\ell^F(p)$ and so ${}_3\ell^F(p)$ is solid.

Also by **Lemma,** it follows that the space ${}_3\ell^F(p)$ is monotone. ■

**Theorem 3.** *The space* ${}_3\ell^F(p)$ *is a sequence algebra..*

**Proof.** Let $\langle X_{nkl} \rangle, \langle Y_{nkl} \rangle \in {}_3\ell^F(p)$

Then $\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left[\bar{d}(X_{nkl} \otimes Y_{nkl}, \bar{0})\right]^{p_{nkl}}$

$$\leq \sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left[\bar{d}(X_{nkl}, \bar{0})\right]^{p_{nkl}} \sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left[\bar{d}(Y_{nkl}, \bar{0})\right]^{p_{nkl}} < \infty$$

Thus $\langle X_{nkl} \otimes Y_{nkl} \rangle \in {}_3\ell^F(p)$ and so ${}_3\ell^F(p)$ is a sequence algebra. ■

**Theorem 4.** *The space* ${}_3\ell^F(p)$ *is not convergence free in general.*

**Proof.** The result follows from the following example.

**Example1.** Let $p_{nk} = \begin{cases} 3, \text{ if } & n=k=l \\ 2, \text{ if } & n=k+l, l=n+k, k=n+l \\ \dfrac{1}{nkl}, \text{ otherwise} & \end{cases}$

We define the sequence $\langle X_{nkl} \rangle$ as follows:

$$X_{nkl} = \bar{0}, \quad \text{for } n \neq k \neq l.$$

Otherwise

$$X_{nkl}(t) = \begin{cases} 1 + nklt, & \text{for } -\dfrac{1}{nkl} \leq t \leq 0 \\ 1 - nklt, & \text{for } 0 \leq t \leq \dfrac{1}{nkl} \\ 0, & \text{otherwise} \end{cases}$$

Then $\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left[\bar{d}(X_{nkl}, \bar{0})\right]^{q_{nkl}}$

$$= \sum_{n=1}^{\infty}\left(\frac{1}{n^3}\right)^3 + \sum_{k=1}^{\infty}\sum_{k=1}^{\infty}\left(\frac{1}{n^2k}\right)^2 + \sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{1}{n^2l}\right)^2 + \sum_{k=1}^{\infty}\sum_{k=1}^{\infty}\left(\frac{1}{nk^2}\right)^2 < \infty.$$

Hence $\langle X_{nkl} \rangle \in {}_3\ell^F(p)$

Let us consider the sequence $\langle Y_{nkl} \rangle$ defined as follows:

$$Y_{nkl} = \bar{0}, \quad \text{for } n \neq k \neq l.$$

Otherwise

$$Y_{nkl}(t) = \begin{cases} \dfrac{1}{2}\left(1 + \dfrac{t}{nkl}\right), & \text{for } -nkl \leq t \leq nkl \\ \left(2 - \dfrac{t}{nkl}\right), & \text{for } nkl \leq t \leq 2nkl \\ 0, & \text{otherwise} \end{cases}$$

Then

$$\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left[\bar{d}(Y_{nkl}, \bar{0})\right]^{p_{nkl}} = \sum_{n=1}^{\infty}(2n^3)^3 + \sum_{k=1}^{\infty}\sum_{k=1}^{\infty}(2n^2k)^2 + \sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(2n^2l)^2 + \sum_{k=1}^{\infty}\sum_{k=1}^{\infty}(2nk^2)^2 = \infty.$$

Hence $\langle Y_{nkl} \rangle \notin {}_3\ell^F(p)$.

So the space ${}_3\ell^F(p)$ is not convergence free.

■

**Theorem 5.** *The space* ${}_3\ell^F(p)$ *is not symmetric*

**Proof .** The proof follows from the following example.

**Example2.** Let $p_{nkl} = \begin{cases} 1, & \text{if } n=k=l \\ 2, & \text{otherwise} \end{cases}$

We consider the sequence $\langle X_{nkl} \rangle$ defined by:

$$X_{nnn}(t) = \begin{cases} 1 + \sqrt{3}nklt, & \text{for } -\dfrac{1}{\sqrt{3}nkl} \leq t \leq 0 \\ 1 - \sqrt{3}nklt, & \text{for } 0 \leq t \leq \dfrac{1}{\sqrt{3}nkl} \\ 0, & \text{otherwise} \end{cases}$$

and $X_{nkl} = \bar{0}$, *otherwise*

Then

$$\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left[\bar{d}(X_{nkl}, \bar{0})\right]^{p_{nkl}} = \sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{3n^3}}\right) + \sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\left(\frac{1}{\sqrt{3n^2k}}\right)^2$$

$$+\sum_{n=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{1}{\sqrt{3}n^2 l}\right)^2+\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\left(\frac{1}{\sqrt{3}nk^2}\right)^2<\infty.$$

Thus $\langle X_{nkl}\rangle \in {}_3\ell^F(p)$.

Let us consider the rearrangement $(Y_{nkl})$ of $(X_{nkl})$ defined as follows:

$$Y_{nkl}=\begin{cases} X_{nnl}, & if \;\; l\neq k=n \\ X_{n11}, & if \;\; l=k=n \\ X_{nkn}, & if \;\; n=l\neq k \\ X_{nkk}, & if \;\; n\neq l=k\neq 1 \\ \bar{0}, & otherwise \end{cases}$$

Then

$$\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left[\bar{d}(Y_{nkl},\bar{0})\right]^{p_{nkl}} = \sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{3n}}\right)+\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\left(\frac{1}{\sqrt{3}n^2k}\right)^2+$$

$$\sum_{n=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{1}{\sqrt{3}n^2 l}\right)^2+\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\left(\frac{1}{\sqrt{3}nk^2}\right)^2=\infty.$$

$$\Rightarrow \langle Y_{nkl}\rangle \notin {}_3\ell^F(p).$$

Hence the space ${}_3\ell^F(p)$ is not symmetric. ■

**Theorem 6.** *If* $0<p_{nkl}<q_{nkl}\leq \sup_{nkl} q_{nkl}$, then ${}_3\ell^F(p)\subset {}_3\ell^F(q)$ and the inclusion is proper.

**Proof.** Let $\langle X_{nkl}\rangle \in {}_3\ell^F(p)$.

$$\Rightarrow \sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left[\bar{d}(X_{nkl},\bar{0})\right]^{p_{nkl}}<\infty.$$

Then there exists $n_0, k_0, l_0 \in N$ such that $\left[\bar{d}(X_{nkl},\bar{0})\right]^{p_{nkl}}<1$, for all $n\geq n_0$ or $k\geq k_0$ or $l\geq l_0$ or for all.

$\Rightarrow \left[\bar{d}(X_{nkl},\bar{0})\right]^{q_{nkl}}<\left[\bar{d}(X_{nkl},\bar{0})\right]^{p_{nkl}}$ for all $n\geq n_0$ or $k\geq k_0$ or $l\geq l_0$.

$$\Rightarrow \sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left[\bar{d}(X_{nkl},\bar{0})\right]^{q_{nkl}}<\infty.$$

$\Rightarrow \langle X_{nkl}\rangle \in {}_3\ell^F(q)$.

Hence ${}_3\ell^F(p)\subset {}_3\ell^F(q)$.

To prove the inclusion to be proper, we consider the following example.

**Example 3.** Let $q_{nkl}=\begin{cases} 3+\frac{1}{n}, & if \quad n=k=l \\ 2, & otherwise \end{cases}$

and $p_{nkl}=\begin{cases} 2, & if \quad n=k=l \\ 1, & otherwise \end{cases}$

We consider the sequence $(X_{nk})$ defined as follows:

$$X_{nkl}=\bar{0}, \quad \text{for } n\neq k\neq l.$$

Otherwise

$$X_{nkl}(t)=\begin{cases} 1+nklt, & \text{for } -\frac{1}{nkl}\leq t\leq 0 \\ 1-nklt, & \text{for } 0\leq t\leq \frac{1}{nkl} \\ 0, & otherwise \end{cases}$$

Then $\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left[\bar{d}(X_{nkl},\bar{0})\right]^{q_{nkl}} = \sum_{n=1}^{\infty}\left(\frac{1}{n^3}\right)^{3+\frac{1}{n}}+$

$$\sum_{k=1}^{\infty}\sum_{k=1}^{\infty}\left(\frac{1}{n^2k}\right)^2+\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{1}{n^2 l}\right)^2+\sum_{k=1}^{\infty}\sum_{k=1}^{\infty}\left(\frac{1}{nk^2}\right)^2<\infty$$

$$\Rightarrow \langle X_{nkl}\rangle \in {}_3\ell^F(q).$$

But $\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left[\bar{d}(X_{nkl},\bar{0})\right]^{p_{nkl}} = \sum_{n=1}^{\infty}\left(\frac{1}{n^3}\right)^2+$

$$\sum_{k=1}^{\infty}\sum_{k=1}^{\infty}\left(\frac{1}{n^2k}\right)+\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left(\frac{1}{n^2 l}\right)+\sum_{k=1}^{\infty}\sum_{k=1}^{\infty}\left(\frac{1}{nk^2}\right)=\infty.$$

$$\Rightarrow \langle X_{nkl}\rangle \notin {}_3\ell^F(p).$$

Hence the inclusion is proper. ■

## IV. CONCLUSION

Convergence theory is used as a basic tool in, measure spaces, sequences of random variables, information theory etc. We have introduced several notions from classical sequence spaces and fuzzy sequence spaces to the new setting of the classes of fuzzy real-valued multiple sequence spaces. Applying metric in the classes of multiple sequences of fuzzy real numbers, some important results are introduced. Although we prove our results only for triple sequences, but all these results remain true for d-multiple sequences as well.

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